MATHEMATICAL MODELS AND CRITERIA OF DECISIONMAKING IN IMAGES IDENTIFICATION

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ABSTRACT

This paper is devoted to one of the most important practical case in image processing application - recognition / identification of the images which, having partly known structure, have unknown, changing in time spatial position. In this case identification process can be separated in two parts being carried out in succession: detection (recognition) and tracing. Mathematical formalization of the task using specific matrices and vectors is suggested and some possible solutions of the problem are discussed.

INTRODUCTION

The problem to be examined can be described for instance as follows: There is a moving endless tape with a periodically repeated drawing, which can have determined or random structure with some unknown a priori defects. The surface of the tape is observed by line TV cameras or photoelectric cells, arranged across the tape. Velocity and position of the tape may be changed accidentally. TV camera has a fixed position and is connected to a computer.

Also there is an image obtained in advance which represents the repeating drawing (one complete period) of the desirable quality and is inputted into computer. This image is referred to as referring image (RI). The problem consists of three tasks [1]:

1. To determine whether the outputs of row TV camera or line photoelectric cells belongs to the RI (identification process)?
2. To determine its position in the RI.
3. To begin and to hold tracing the drawing on the moving tape.

Each task of the problem can be applied to corresponding particular section of Statistical Theory.

The first task can be solved by using the Detection Theory, based on the Statistical Hypotheses Testing Theory, the second and the third tasks are the subjects of the

Parameter Estimation Theory and Filtering Theory, respectively. Notation and geometry representation is shown on fig. 1.

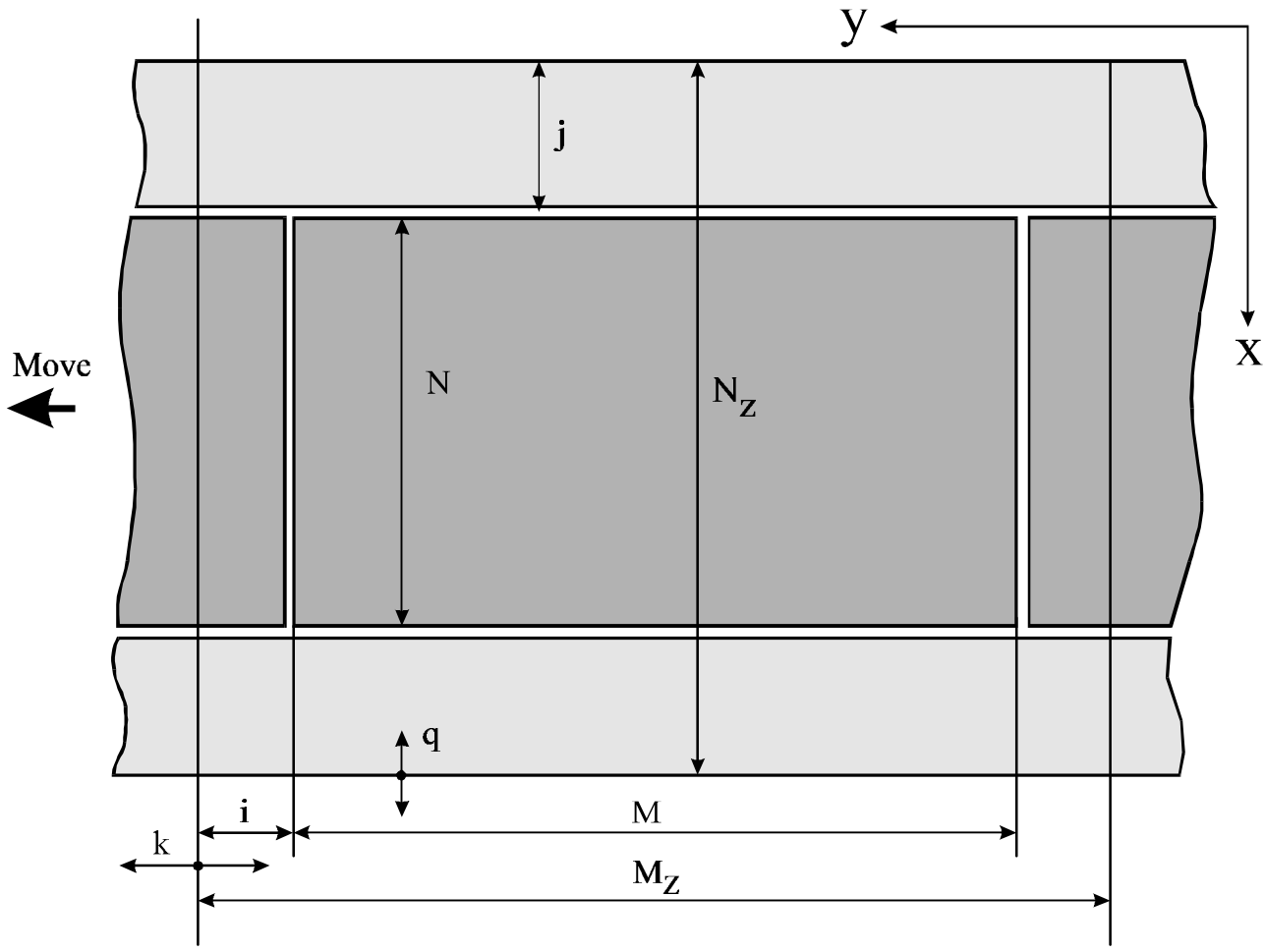


Figure 1. Geometry representation of identification object.

SECTION I. MOVING WITHOUT ROTATION

1. FORMALIZATION OF THE PROBLEM

1.1. REFERENCE ARRAY

Let us suppose that any image is represented by (N_z x M_z) array $\mathbf{Z}_0 \in \mathbf{R}_+$, where $\mathbf{R}_+$ is the subset of the all real positive numbers (including 0), $\mathbf{R}_+ \oplus \mathbf{R}_- = \mathbf{R}$. This array corresponds to the image of the desirable quality.

It is used for forming the reference (N x M) - array $\mathbf{Y}_0$, $N < N_z$, $M < M_z$. This forming can be done by using special selecting (N x N_z) - matrix $\mathbf{V}_n$, which realises selection of the elements $\{\mathbf{Y}_0\}_{nm}$ in succession along axis X (with n = variable), and (M x M_z) - matrix $\mathbf{V}_m$ for the elements in succession along axis Y (with m = variable).

So,

$$\mathbf{Y}_0 = \mathbf{V}_n \, \mathbf{Z}_0 \, \mathbf{V}_m^T,$$

where

$$n = j, \ldots, j + N, \qquad j = 1, \ldots, N_z - N,$$

$$m = i, \ldots, i + \grave{I}, \qquad i = l, \ldots, M_z - \grave{I}.$$

In reality array $\mathbf{Z}_0 = \mathbf{Z}_{00} + \mathbf{N}_0$, where $\mathbf{Z}_{00}$ represents noiseless image, and $\mathbf{N}_0$ - additive noise, but in this work we will use for computer modeling just $\mathbf{Z}_0$ as reference array.

Components of the matrices $\mathbf{V}_n$ and $\mathbf{V}_m$ can be exposed to determined and random shifts represented by the shift matrices $\mathbf{C}_x^q$ and $\mathbf{C}_y^k$ which dimensions are (N_z x N_z) and (M_z x M_z) respectively; q = qd + qr, k = kd + kr, where qd and kd are the determined positive integer variables; qr and kr are the random (positive or negative) integer variables. These integer variables should satisfy the following conditions:

$$0 \le q \le (N_z - N - 1),$$

$$0 \le k \le (M_z - M - 1).$$

So that:

$$\mathbf{Y}_0 = \mathbf{V}_n \, \mathbf{C}_x^q \, \mathbf{Z}_0 \, \mathbf{C}_y^{-k} \mathbf{V}_m^T. \qquad (1)$$

The structures of the matrices **V** and **C** can be as follows:

$$\mathbf{V}_n = \mathbf{V}_x \, \mathbf{C}_x^n,$$

where $\mathbf{V}_x = [\mathbf{I}_N : \mathbf{O}_{N,\, Nz - N}]$ - the selecting matrix as n = l (see fig. 2);

$\mathbf{I}_N$ - the identity (N x N) - matrix;

$\mathbf{O}_{N,\, Nz - N}$ - the zero-component (N x (N_z - N)) - matrix.

$$\mathbf{V}_X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{I}_5 \qquad \mathbf{O}_{5,3}$$

Figure 2. View of (5 x 8) - matrix $\mathbf{V}_x$.

$$\tilde{\mathbf{N}}_x = [\mathbf{i}_{Nz}, \mathbf{i}_1, \ldots, \mathbf{i}_{Nz-1}], \qquad (2)$$

where $\mathbf{i}_{Nj}$ - (N_z x 1) - vector, having 1 at the j-th position and 0 at the others.

i_{N5} - vector

$$\mathbf{C_x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5-th position

Figure 3. View of (5 x 5) - matrix $\tilde{\mathbf{N}}_x$.

Matrices $\mathbf{V}_m$ and $\mathbf{C}_m$ have the similar structures:

$$\mathbf{V}_m = \mathbf{V}_y\, \mathbf{C}_y^{\,m},$$

$$\mathbf{V}_y = [\mathbf{I}_M : \mathbf{O}_{M,\, Mz-M}],$$

$$\tilde{\mathbf{N}}_y = [\mathbf{i}_{Mz}, \mathbf{i}_1, \ldots, \mathbf{i}_{Mz-1}].$$

Thus

$$\mathbf{Y}_0 = \mathbf{V}_x\, \mathbf{C}_x^{\,q+n}\, \mathbf{Z}_0\, \mathbf{C}_y^{\,-(k+m)} \mathbf{V}_y^{\,T}.$$

$$S_{\mathbf{0}} = \begin{bmatrix} s_{11} & s_{21} & s_{31} & s_{41} & s_{51} & s_{61} & s_{71} & s_{81} & s_{91} & s_{101} \\ s_{12} & s_{22} & s_{32} & s_{42} & s_{52} & s_{62} & s_{72} & s_{82} & s_{92} & s_{102} \\ s_{13} & s_{23} & s_{33} & s_{43} & s_{53} & s_{63} & s_{73} & s_{83} & s_{93} & s_{103} \\ s_{14} & s_{24} & s_{34} & s_{44} & s_{54} & s_{64} & s_{74} & s_{84} & s_{94} & s_{104} \\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55} & s_{65} & s_{75} & s_{85} & s_{95} & s_{105} \\ s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & s_{66} & s_{76} & s_{86} & s_{96} & s_{106} \\ s_{17} & s_{27} & s_{37} & s_{47} & s_{57} & s_{67} & s_{77} & s_{87} & s_{97} & s_{107} \\ s_{18} & s_{28} & s_{38} & s_{48} & s_{58} & s_{68} & s_{78} & s_{88} & s_{98} & s_{108} \end{bmatrix}$$

$\mathbf{Y}_0$ - matrix

Figure 4. View of (4 x 5) - matrix $\mathbf{Y}_0$.

For simplicity we shall consider later that n and m are included in qd and kd respectively.

It is useful to notice that $\mathbf{VV}^T = \mathbf{I}$ and $\mathbf{C}^T = \mathbf{C}^{-1}$, where $\mathbf{C}^T$ is the transpose of $\mathbf{C}$, i.e. $\mathbf{C}$ the is an orthogonal matrix.

1.2. SAMPLE ARRAY

The random sample array to be identified can be represented as

$$\mathbf{Z} = (\mathbf{Z}_0 + \mathbf{Z}_d)\mathbf{A} + \mathbf{N}, \qquad (3)$$

where $\mathbf{A} = \mathrm{diag}\,[a_1, \ldots, a_{Mz}]$ - matrix of a random variables, $a_i \in \mathbf{R}_+$, $\mathbf{Z}_d \in \mathbf{R}$ - $(N_z \times M_z)$ - matrix of (random) errors, $\mathbf{N}$ - a sample noise matrix.

Analogously (1) for the testing sample array we can write:

$$\mathbf{Y} = \mathbf{V}_x\, \mathbf{C}_x^{\,s}\, \mathbf{Z}\, \mathbf{C}_y^{\,-t}\, \mathbf{V}_y^{\,T},$$

where s = sd + sr, t = td + tr; sd and td are the determined positive integer variables (including n and m), sr and tr - random integer variables (positive or negative),

$$0 \le s \le (N_z - N - 1),$$

$$0 \le t \le (M_z - M - 1).$$

Each time for the identification we have only the column vector $\mathbf{y}_j \in \mathbf{Y}$:

$$\mathbf{y}_j = \mathbf{Y}\, \mathbf{i}_{Mj} = \mathbf{V}_x\, \mathbf{C}_x^{\,s}\, \mathbf{Z}\, \mathbf{C}_y^{\,-t}\, \mathbf{i}_{Mj}\,, \qquad (4)$$

where (M x l) - vector $\mathbf{y}_j$ and $(M_z \times l)$ - vector $\mathbf{i}_{Mj}$ are analogous to vectors, used in (2). So far as $\mathbf{C}_y^{\,-t}\, \mathbf{i}_{Mj} = \mathbf{C}_y^{\,-t-j+1}\, \mathbf{i}_{M1}$ (see fig. 5) we can write

$$\mathbf{y}_j = \mathbf{Y}\, \mathbf{i}_{Mj} = \mathbf{V}_x\, \mathbf{C}_x^{\,s}\, \mathbf{Z}\, \mathbf{C}_y^{\,-t1}\, \mathbf{i}_{M1}\,, \qquad (5)$$

where tl = t + j - l, $(M_z \times l)$ - vector $\mathbf{i}_{M1}$ has 1 at the first position and 0 at the others.

$$\underbrace{\begin{bmatrix} 0&0&0&1&0\\ 0&0&0&0&1\\ 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&1&0&0 \end{bmatrix}}_{\mathbf{C}_y^{-2}} \underbrace{\begin{bmatrix} 0\\0\\1\\0\\0 \end{bmatrix}}_{\mathbf{i}_{M3}} = \underbrace{\begin{bmatrix} 0&1&0&0&0\\ 0&0&1&0&0\\ 0&0&0&1&0\\ 0&0&0&0&1\\ 1&0&0&0&0 \end{bmatrix}}_{\mathbf{C}_y^{-4}} \underbrace{\begin{bmatrix} 1\\0\\0\\0\\0 \end{bmatrix}}_{\mathbf{i}_{M1}} = \begin{bmatrix} 0\\0\\0\\0\\1 \end{bmatrix}$$

Figure 5. Property of the matrixes multiplication $\mathbf{C}_y^{\,-t}\, \mathbf{i}_{Mj}$.

Identification must be carried out by comparing the components of the vector (5) with the components of the matrix (1). For more detailed analysis let us represent (N x l) - vector $\mathbf{y}_{0i} \in \mathbf{Y}_0$ analogously (5):

$$\mathbf{y}_{0i} = \mathbf{Y}_0\, \mathbf{i}_{Mi} = \mathbf{V}_x\, \mathbf{C}_x^{q}\, \mathbf{Z}\, \mathbf{C}_y^{-k1} \mathbf{i}_{M1} \quad , \qquad (6)$$

where kl = k + i - l .

2. IDENTIFICATION OF THE SAMPLE VECTORS

2.1. MATHEMATICAL APPROACH

In general to solve the problem is to test two statistical hypotheses: "zero' hypothesis

$$H_0: \quad \mathbf{y}_j \in \mathbf{Y}_0 \qquad \text{as} \qquad j = l, ..., \grave{I} , \qquad (7)$$

against alternative hypothesis

$$H_1: \quad \mathbf{y}_j \notin \mathbf{Y}_0 \qquad \text{as} \qquad j = l, ..., M. \qquad (8)$$

This case is the case of composite hypotheses testing (because some unknown parameters are presented). We can consider the case as multialternative testing - in this case - verification of (M + l) hypotheses:

"zero" hypothesis

$$H_0: \qquad \mathbf{y} = \mathbf{N}, \qquad (7)$$

against M alternative hypotheses

$$H_i: \qquad \mathbf{y} = \mathbf{y}_{0i} + \mathbf{N}_i \qquad \text{as} \qquad i = l, ..., M. \qquad (8)$$

Hypothesis H_0 means that no image is in the TV camera output signal. This case is out of any practical interest. In fact we deal with the problem of distinguishing of M non orthogonal vector signals. This problem is rather complicated and by now has no complete theoretical solution in general. But in many applications we deal with the similar tasks, therefore some theoretical (for particular cases) and heuristic results are known.

The multialternative testing approach is very close to the estimation theory. Therefore solving the distinguishing problem we simultaneously can obtain a result for the estimation part of it. It is clear that the result depends on the choice of a criterion.

As we can combine detection and estimation tasks it is convenient to choose Maximum Likelihood ratio Method.

In accordance with this method we have to choose the maximum value of Likelihood ration obtained for the all comparing vectors:

$$\Lambda_i(\mathbf{y}) \to \max \quad \text{as} \quad i = l,..., M,$$

or

$$\ln \Lambda_i(\mathbf{y}) \to \max \quad \text{as} \quad i = l, ..., M,$$

where the Likelihood ratio is

$$\Lambda_i(\mathbf{y}) = [\sup L_y(i)] / [\sup L_y(0)].$$

$L_y(0)$ and $L_y(i)$ - likelihoods, corresponding to "zero" hypothesis and M alternative hypotheses, respectively.

The value of the variable i, corresponding to Likelihood ratio maximum, gives the evaluation of the image vector (which at the moment has been received from line TV camera) number.

When array dimension M is too large for real time processing, it is possible to restrict the maximization range assuming $i = M_1, ..., M_2$, where $[M_1, M_2] \in [1, Ì]$, forming in such a way a "recognition window". This window can be immovable since sample vector is "moving" (changing in accordance with a tape movement).

To use the optimal (in sense of Maximum Likelihood ratio) rule of decision it is necessary to know statistical characteristics of the random variables. As was noted above the structure of an image can be both determined and random one. Theoretically it is possible to calculate all the characteristics of the reference image and in such a way to determine reference array $\mathbf{Z}_0$, especially for determined (geometrical) drawing [2]. But such a work is rather complicated and needs a very high level accuracy of the joint positioning of the tape and TV camera. Practically reference array $\mathbf{Z}_0$, can be obtained by averaging (or selecting) some arrays corresponding to the images of the desirable quality.

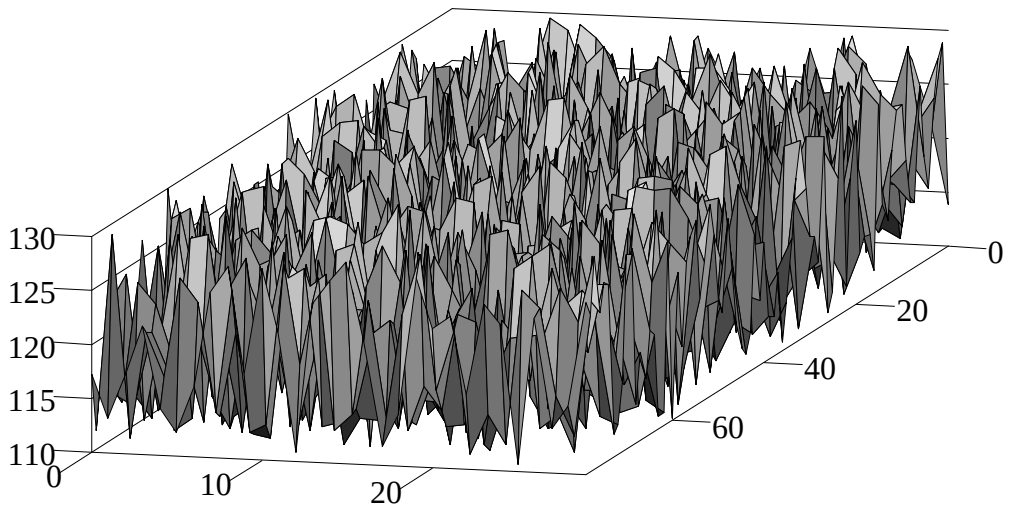
It is useful to note that in the discussing problem we deal with a multidimensional cyclostationary process which distribution depends on the group of random parameters with specified distributions.

2.2. HEURESTIC APPROACH

If statistical characteristics of the random variables ($à_i$, qr, kr, sr, tr) are unknown and the sample size is not enough to estimate them with required accuracy (in the cases of parametric or nonparametric a priori uncertainties) it is impossible to use any constructive mathematical methods to obtain the solution of the problem.

To solve the problem at this rate it is possible to apply a heuristic approach. Below some possible criteria are discussed in brief.

On figures 6 and 7 you can see of view reference matrix for computer modeling.



Zo

Figure 6. View the example of reference array with equal randomize of numbers (root mean square this reference array is 5.8).

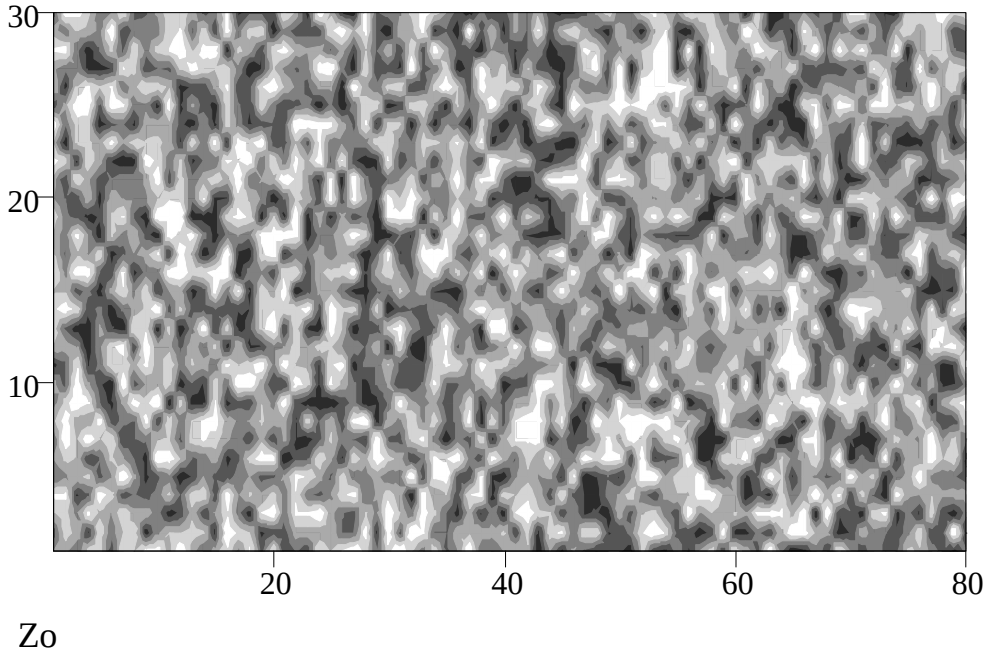


Figure 7. View the example of reference array with equal randomize of numbers as grayscale image.

2.3. SOME CRITERIA DISCUSSION

Let's look at the problem from point of view of definition of distance between vectors with use of the following criteria.

1. Minimum norm (or square norm) of the difference vector. This criterion can be written as

$$|\mathbf{y}_j - \mathbf{y}_{0i}| \to \min \quad \text{as } i = j,$$

or

$$|\mathbf{y}_j - \mathbf{y}_{0i}|^2 \to \min \quad \text{as } i = j. \qquad (9)$$

Both forms of the criterion are equal but for discussion it is more convenient to use the second one.

For (9) we can write:

$$|\mathbf{y}_j - \mathbf{y}_{0i}|^2 = |\mathbf{y}_j|^2 + |\mathbf{y}_{0i}|^2 - 2\mathbf{y}_j^T\mathbf{y}_{0i}\,. \qquad (10)$$

Using (5) and (6), obtain

$$|\mathbf{y}_j|^2 = (\mathbf{V}_x\,\mathbf{C}_x^{\,s}\,\mathbf{Z}\,\mathbf{C}_y^{\,-t1}\mathbf{i}_{M1})^T\,(\mathbf{V}_x\,\mathbf{C}_x^{\,s}\,\mathbf{Z}\,\mathbf{C}_y^{\,-t1}\mathbf{i}_{M1}) =$$

$$= \mathrm{Sp}\,(\mathbf{y}_{(s+1),\,(t1+1)}\,\mathbf{y}_{(s+1),\,(t1+1)}^T) = |\mathbf{y}_{(s+1),\,(t1+1)}|^2\,, \qquad (11)$$

where Sp(.) denotes trap of a matrix, $\mathbf{y}_{(s+1),\,(t1+1)}$ - (N x l) - vector, the number of the first component of which (inside array **Z**) is (s + l), (tl + l).

Analogously

$$\left|\mathbf{y}_{0i}\right|^2 = \left|\mathbf{y}_{0(q+1),\,(k1+1)}\right|^2, \qquad (12)$$

and

$$\mathbf{y}_j^T\mathbf{y}_{0i} = \mathbf{y}_{(s+1),\,(t1+1)}^T\,\mathbf{y}_{0(q+1),\,(k1+1)}. \qquad (13)$$

Since

$$\mathbf{y}_{0i} = \mathbf{z}_{00(q+1),\,(k1+1)} + \mathbf{n}_{0(q+1),\,(k1+1)},$$

$$\mathbf{y}_j = (\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)})\,a_{(t1+1)} + \mathbf{n}_{(s+1),\,(t1+1)}.$$

we can write:

$$\left|\mathbf{y}_{0i}\right|^2 = \left|\mathbf{z}_{00(q+1),\,(k1+1)}\right|^2 + \left|\mathbf{n}_{0(q+1),\,(k1+1)}\right|^2 + \\ + 2\,\mathbf{z}_{00(q+1),\,(k1+1)}^T\,\mathbf{n}_{0(q+1),\,(k1+1)}. \qquad (14a)$$

$$\left|\mathbf{y}_j\right|^2 = \left|\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)}\right|^2 a_{(t1+1)}^2 + \left|\mathbf{n}_{(s+1),\,(t1+1)}\right|^2 + \\ + 2\,a_{(t1+1)}(\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)})^T\,\mathbf{n}_{(s+1),\,(t1+1)} = \\ = (\left|\mathbf{z}_{00(s+1),\,(t1+1)}\right|^2 + \left|\mathbf{z}_{d(s+1),\,(t1+1)}\right|^2 + \\ +2\,\mathbf{z}_{00(s+1),\,(t1+1)}^T\,\mathbf{z}_{d(s+1),\,(t1+1)})\,a_{(t1+1)}^2 + \left|\mathbf{n}_{(s+1),\,(t1+1)}\right|^2 + \\ 2\,a_{(t1+1)}(\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)})^T\,\mathbf{n}_{(s+1),\,(t1+1)}, \qquad (14b)$$

$$\mathbf{y}_j^T\mathbf{y}_{0i} = a_{(t1+1)}\,(\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)})^T\,\mathbf{z}_{00(q+1),\,(k1+1)} + \\ + a_{(t1+1)}(\mathbf{z}_{00(s+1),\,(t1+1)} + \mathbf{z}_{d(s+1),\,(t1+1)})^T\,\mathbf{n}_{0(q+1),\,(k1+1)} + \\ + \mathbf{n}_{(s+1),\,(t1+1)}^T\,\mathbf{z}_{00(q+1),\,(k1+1)} + \mathbf{n}_{(s+1),\,(t1+1)}^T\,\mathbf{n}_{0(q+1),\,(k1+1)}. \qquad (14c)$$

Neglecting influences of the noises, we obtain

$$\left|\mathbf{y}_{0i}\right|^2 = \left|\mathbf{z}_{00(q+1),\,(k1+1)}\right|^2, \qquad (15a)$$

$$\left|\mathbf{y}_j\right|^2 = \left|\mathbf{z}_{00(s+1),\,(t1+1)}\right|^2 + \left|\mathbf{z}_{d(s+1),\,(t1+1)}\right|^2 + \\ + 2\,\mathbf{z}_{00(s+1),\,(t1+1)}^T\,\mathbf{z}_{d(s+1),\,(t1+1)}, \qquad (15b)$$

$$\mathbf{y}_j^T\mathbf{y}_{0i} = a_{(t1+1)}(\mathbf{z}_{00(s+1),(t1+1)} + \mathbf{z}_{d(s+1),(t1+1)})^T\,\mathbf{z}_{00(q+1),(k1+1)}\,. \qquad (15c)$$

Evidently, that in the case $\mathbf{z}_{d(s+1),(t1+1)} = 0$ expressions (15b) and (15c) can be written as

$$|\mathbf{y}_j|^2 = |\mathbf{z}_{00(s+1),(t1+1)}|^2\,,$$

$$\mathbf{y}_j^T\mathbf{y}_{0i} = a_{(t1+1)}\,\mathbf{z}_{00(s+1),(t1+1)}^T\,\mathbf{z}_{00(q+1),(k1+1)}\,.$$

Criterion (9) will take on its minimum value (zero, in the last case) if s = q, tl = kl. Influences of noises and errors (matrix $\mathbf{Z}_d$) change the minimum value, moreover the quality function corresponding to criterion (9) becomes not uniminimum one (global minimum determination will not be discussed in this paper).

We can write down statistical average of the matrices product to find out on that variables and how does the criterion depend:

$$E\,[\mathbf{Z}_0^T\mathbf{Z}_0] = E\,[\,(\mathbf{Z}_{00} + \mathbf{N}_0)^T(\mathbf{Z}_{00} + \mathbf{N}_0)\,] = \mathbf{B}_{Z00} + \mathbf{B}_{N0}\,, \qquad (16)$$

$$E\,[\mathbf{Z}^T\mathbf{Z}] = E\,[\,(\,(\mathbf{Z}_{00} + \mathbf{Z}_d)\mathbf{A} + \mathbf{N})^T(\,(\mathbf{Z}_{00} + \mathbf{Z}_d)\mathbf{A} + \mathbf{N})] =$$

$$= E\,[\mathbf{A}^T\mathbf{Z}_{00}^T\,\mathbf{Z}_{00}\mathbf{A}] + E\,[\mathbf{A}^T\mathbf{Z}_d^T\,\mathbf{Z}_d\mathbf{A}] + \mathbf{B}_N\,, \qquad (17)$$

$$E\,[\mathbf{Z}^T\mathbf{Z}_0] = E\,[\,((\mathbf{Z}_{00} + \mathbf{Z}_d)\mathbf{A} + \mathbf{N})^T(\mathbf{Z}_{00} + \mathbf{N}_0)] = E\,[\mathbf{A}^T\mathbf{Z}_{00}^T\,\mathbf{Z}_{00}]\,, \qquad (18)$$

where $\mathbf{B}_{Z00}$, $\mathbf{B}_{N0}$ and $\mathbf{B}_N$ are (M_z x M_z) - matrices of the second-order moments of corresponding random variables. It has been taken into account that different types of the variables are jointly independent.

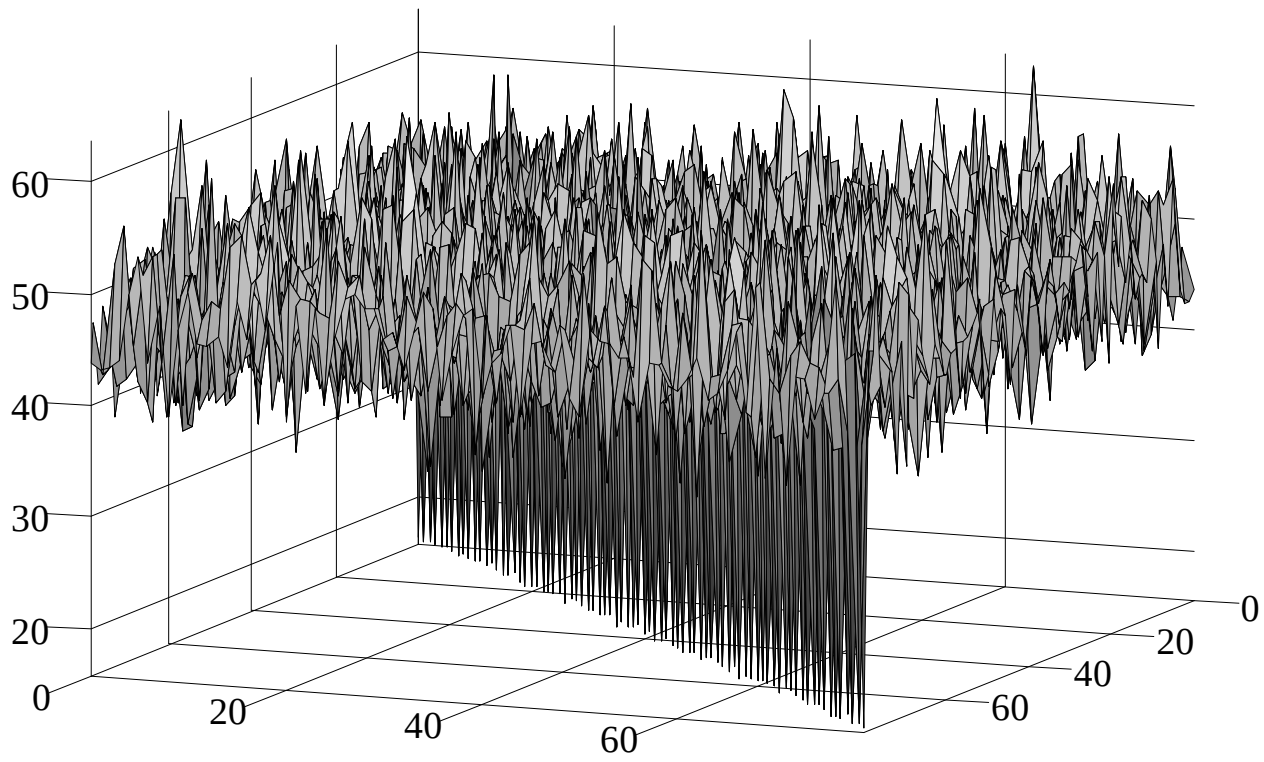
In all criteria we used the random values with normal distribution for modeling sample noise matrix **N**. The mean of this random value was equal to zero.

In the figures enclosed you can see results of computer modeling.

Figure 8 shows the development of magnitudes $|\mathbf{y}_j - \mathbf{y}_{0i}|$, which were averaged by 120 realizations (the X - axes represents the number of a vector from the reference array, which was sequentially compared with all vectors of the sample matrix, on Y - axes - the number of a vector from the sample array). Figure 9 represents values of the diagonal elements of matrix Numb1_1 (see fig. 8).

The figure shows that results of modeling, which obtained by one vector from the array $\mathbf{Z}_0$, will not essentially change if we take any other vector from $\mathbf{Z}_0$ as reference (only for the type of image we consider). For the computing we chose the vector $\mathbf{y}_{0\,40}$.

On the figure 10 the dependence of $|\mathbf{y}_j - \mathbf{y}_{0i}|$ on root mean square deviation of additive noise (Nsigm1 was averaged by 120 realizations) is shown. The figures 11, 12 and 13 we can see the sections Stat1_1 and Stat1_2 of the figure Nsigm1, given the root mean square value of noise 3.0 and 15.0 respectively. On the figures 11 and 12 the dot line shows the examples Samp1_1 and Samp1_2 of the realization $|\mathbf{y}_j - \mathbf{y}_{0i}|$, according to which Stat1_1 and Stat1_2 were calculated. All figures are given in a scale of values $\mathbf{Z}_0$.



Numb1_1

Figure 8.

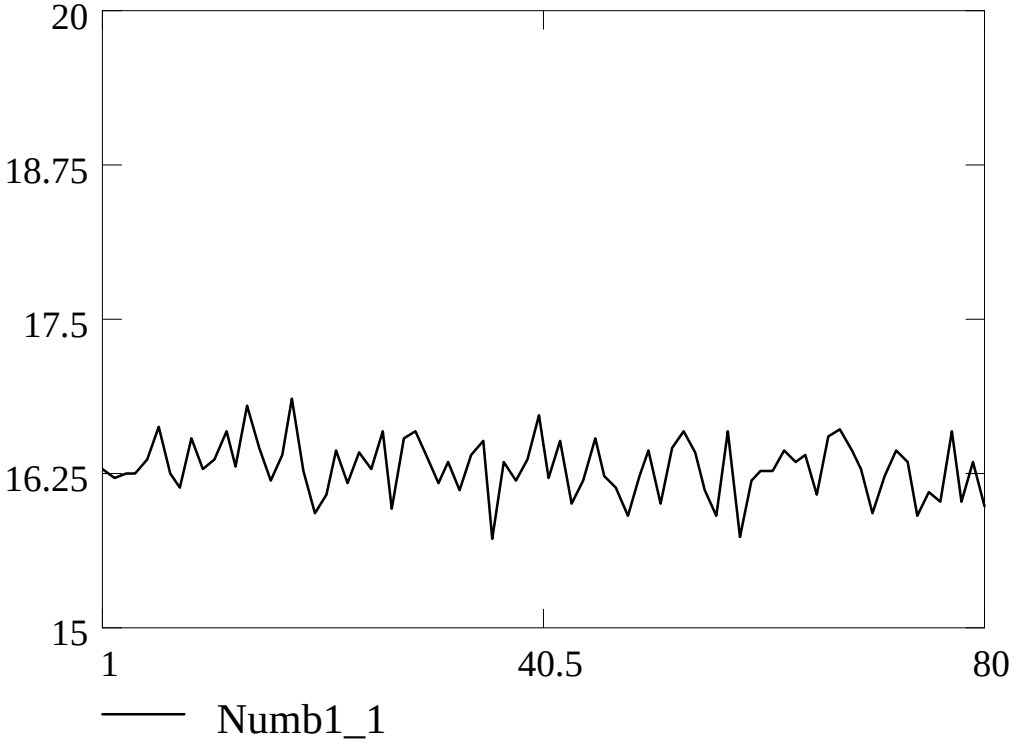


Figure 9.

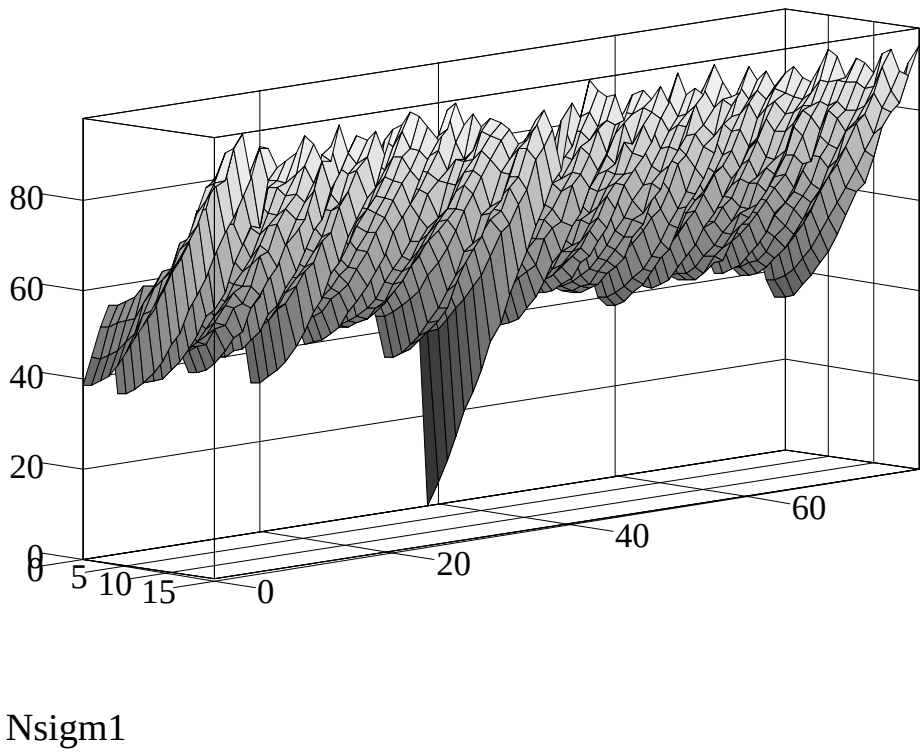


Figure 10.

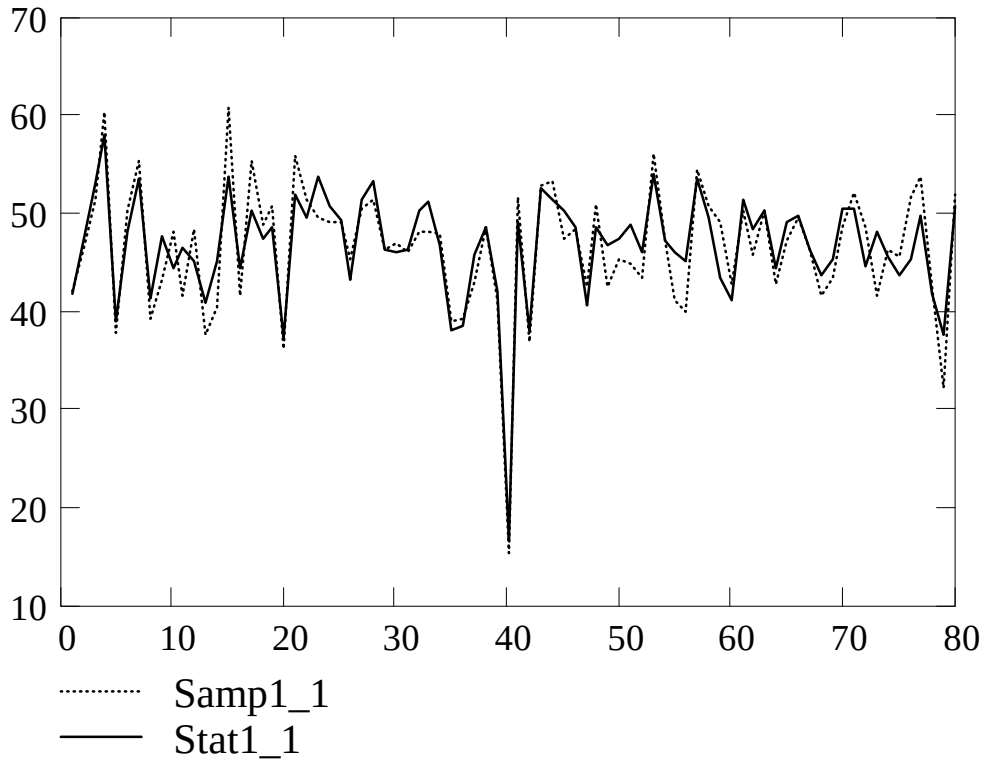


Figure 11.

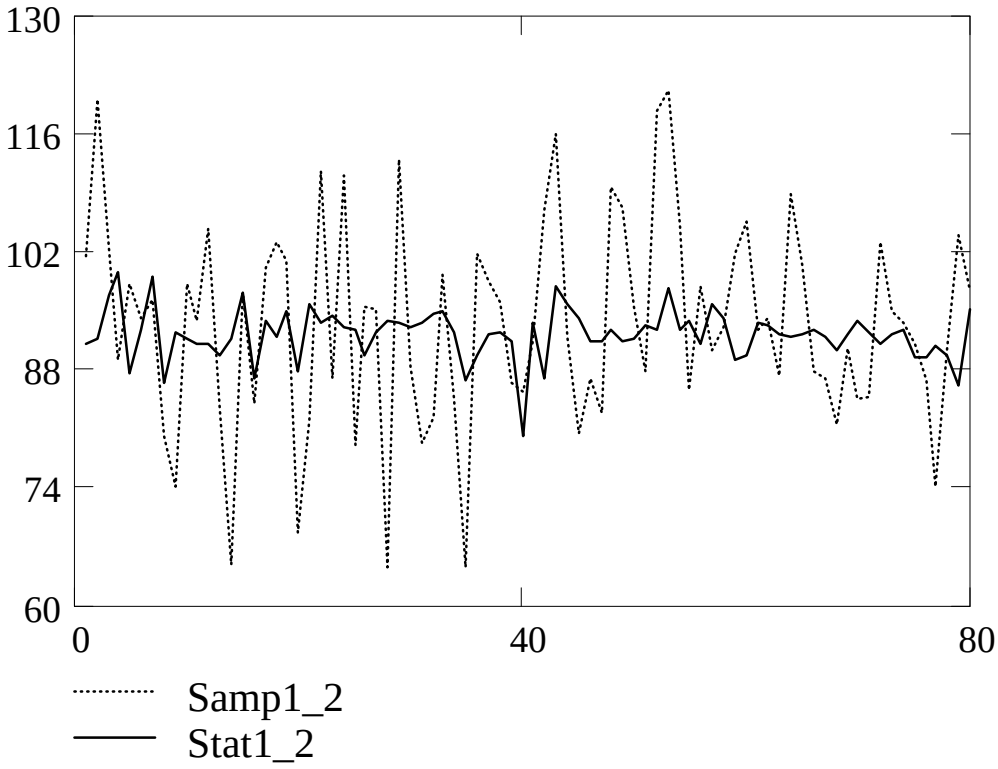


Figure 12.

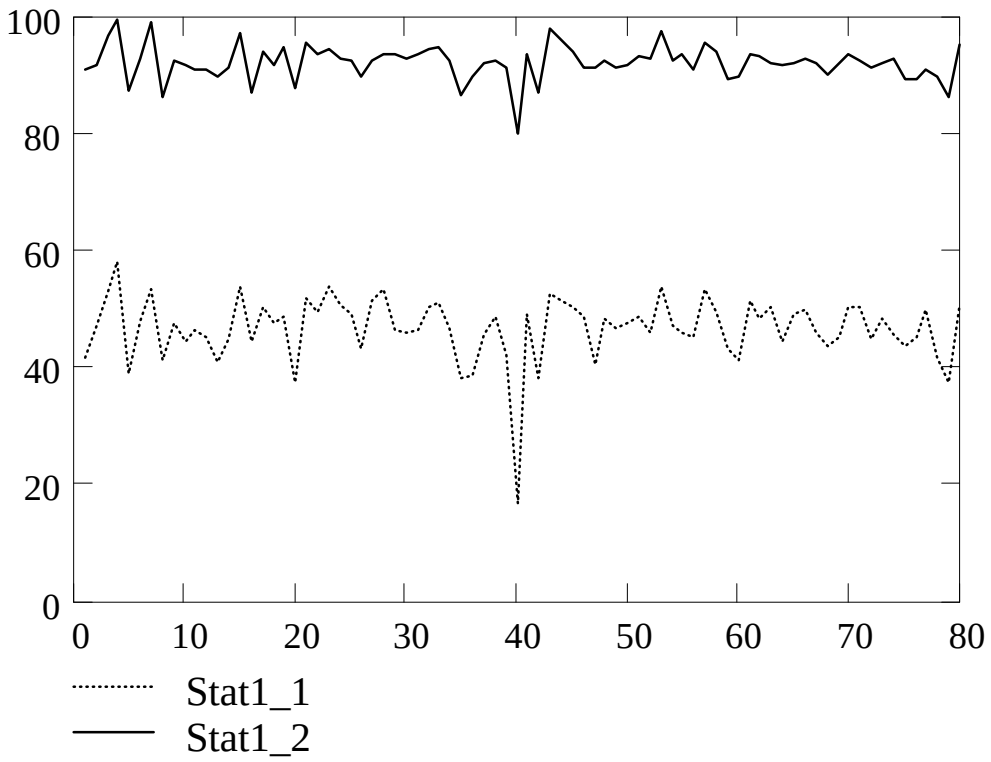


Figure 13.

2. Minimum sum of deviation.

$$|\mathbf{y}_j - \mathbf{y}_{0i}|^T \mathbf{e}_M \rightarrow \min \quad \text{as } i = j\ ,$$

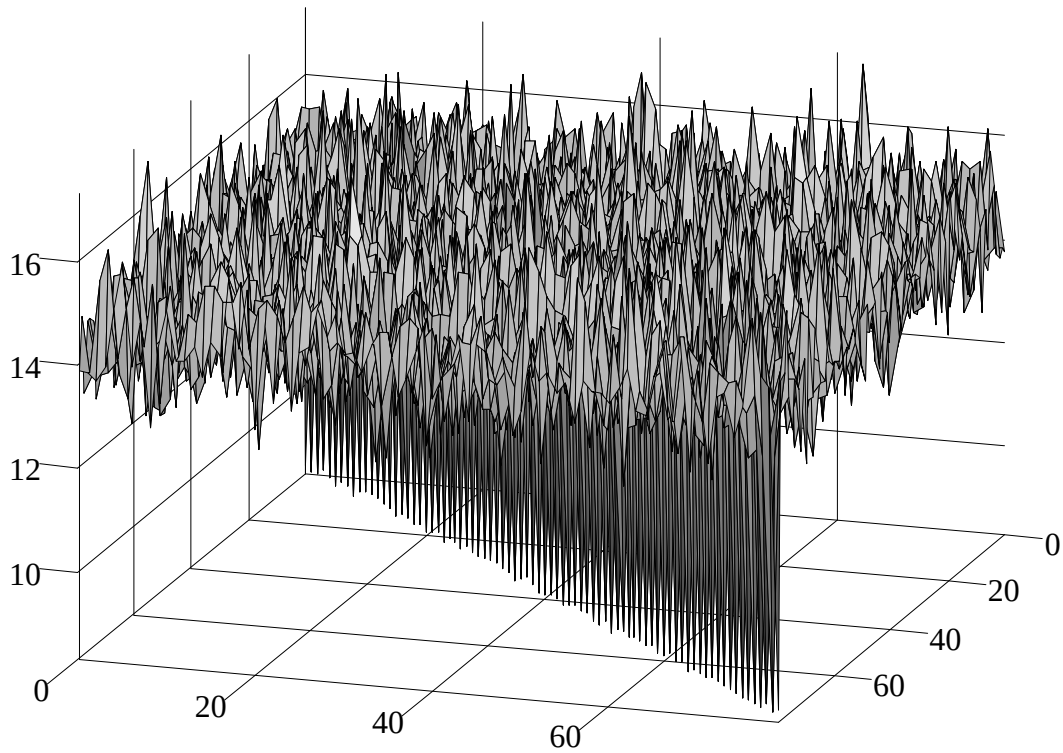
where $\mathbf{e}_M$ - (M x l) - vector all the components of which are equal 1 (summation vector); $|\mathbf{.}|$- denotes a vector of the absolute values of subtraction of the corresponding components of the vectors.

In the figures enclosed you can see results of computer modeling.

Figure 14 shows the development of magnitudes $|\mathbf{y}_j - \mathbf{y}_{0i}|^T \mathbf{e}_M$, which were averaged by 120 realizations (the X - axes represents the number of a vector from the reference array, which was sequentially compared with all vectors of the sample matrix, on Y - axes - the number of a vector from the sample array). Figure 15 represents values of the diagonal elements of matrix Numb2_1 (see fig. 14). The figure shows that results of modeling, which obtained by one vector from the array $\mathbf{Z}_0$, will not essen-

tially change if we take any other vector from $\mathbf{Z}_0$ as reference (only for the type of image we consider). For the computing we chose the vector $\mathbf{y}_{0\,40}$.

On the figure 16 the dependence of $|\mathbf{y}_j - \mathbf{y}_{0i}|^T\mathbf{e}_M$ on root mean square deviation of additive noise (Nsigm2 was averaged by 120 realizations) is shown. The figures 17, 18 and 19 we can see the sections Stat2_1 and Stat2_2 of the figure Nsigm2, given the root mean square value of noise 3.0 and 15.0 respectively. On the figures 17 and 18 the dot line shows the examples Samp2_1 and Samp2_2 of the realization $|\mathbf{y}_j - \mathbf{y}_{0i}|^T\mathbf{e}_M$, according to which Stat2_1 and Stat2_2 were calculated. All figures are given in a scale of values $\mathbf{Z}_0$.



Numb2_1

Figure 14.

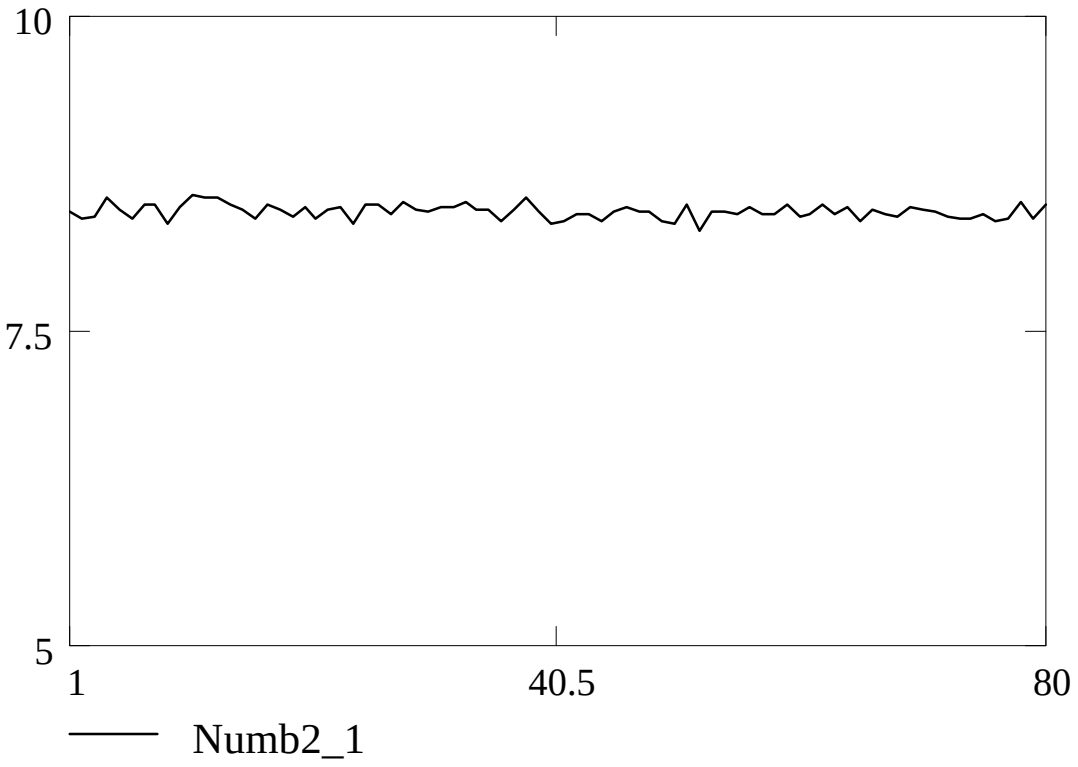


Figure 15.

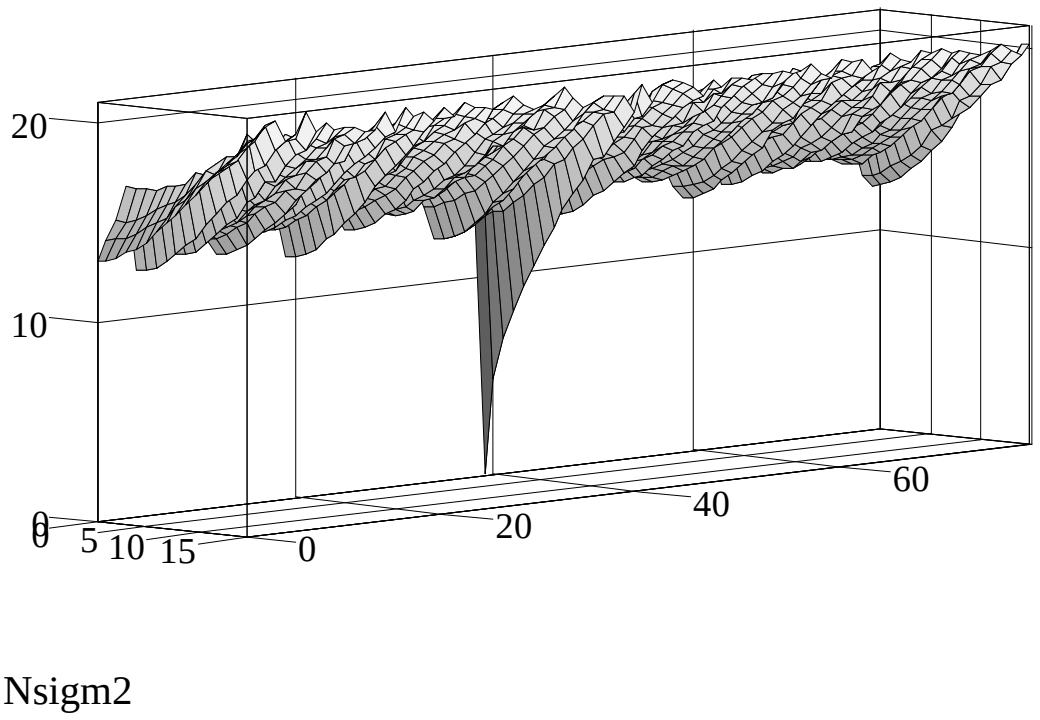


Figure 16.

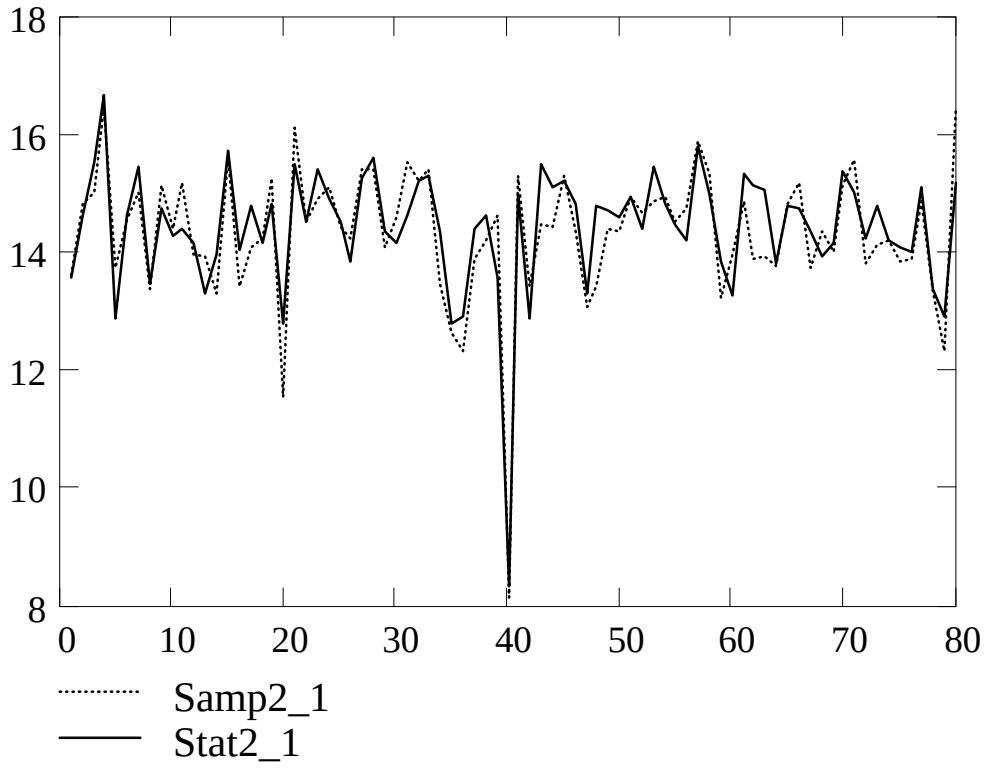


Figure 17.

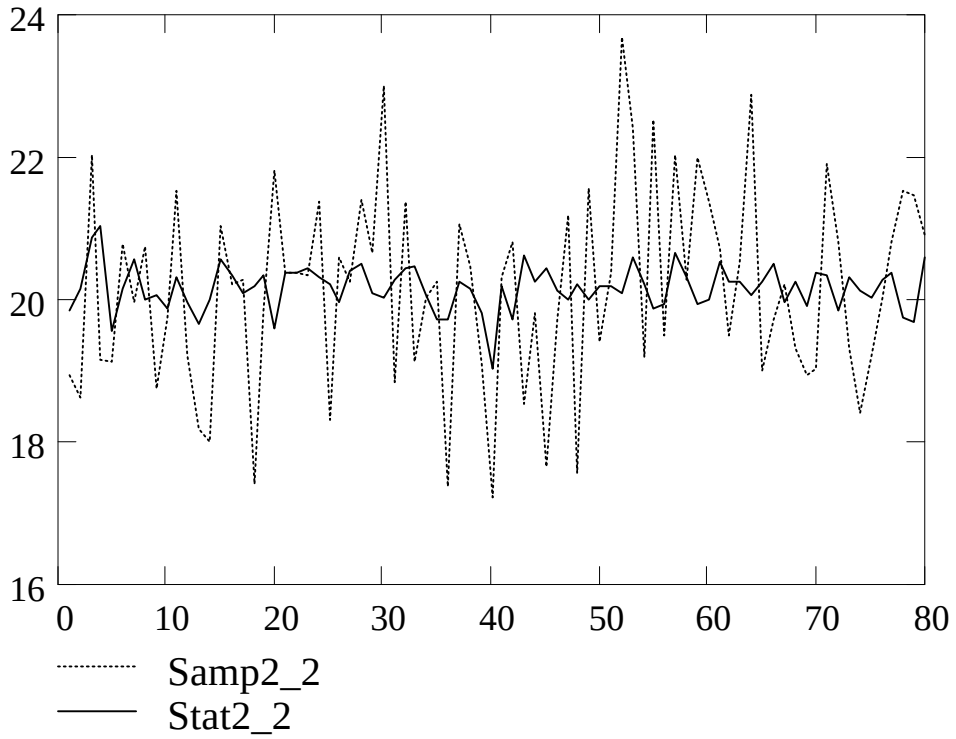


Figure 18.

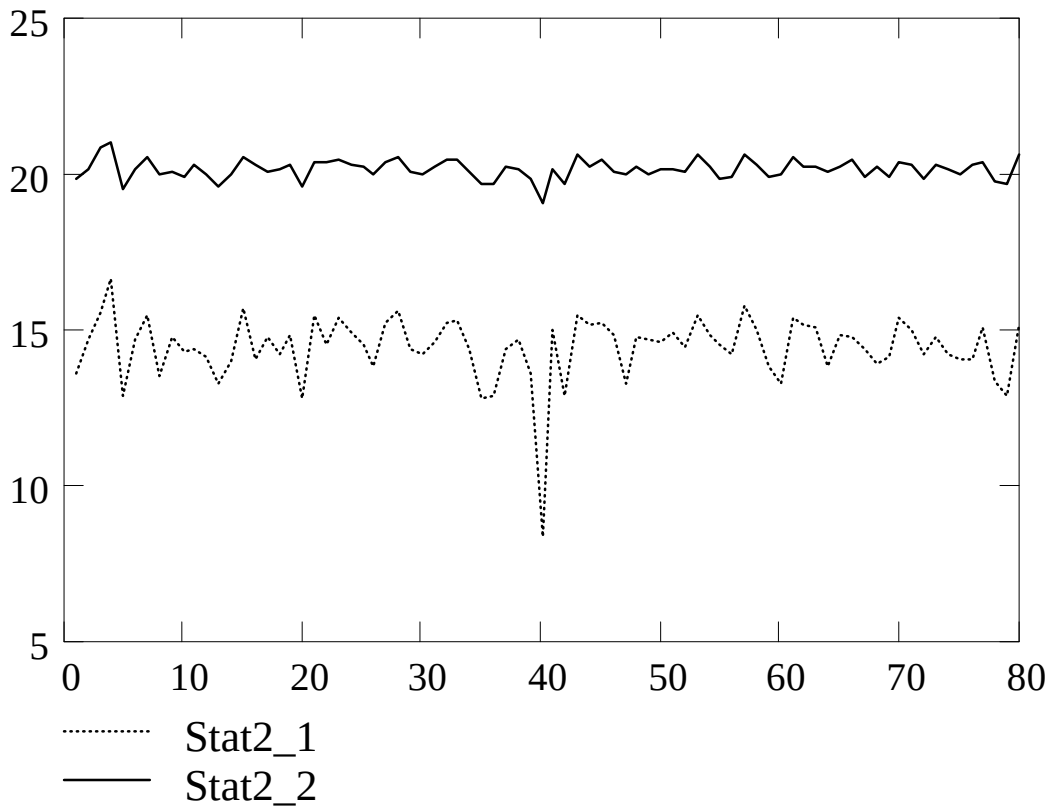


Figure 19.

3. Minimum angle deviation.

$$\cos(\mathbf{L}_{ij}) \rightarrow \max \quad \text{as } i = j\ ,$$

where $\mathbf{L}_{ij}$ - an angle between vectors $\mathbf{y}_j$ and $\mathbf{y}_{0i}$,

$$\cos(\mathbf{L}_{ij}) = \mathbf{y}_j^T\mathbf{y}_{0i} / \mathrm{sqrt}\,(\mathbf{y}_j^T\mathbf{y}_j\ \mathbf{y}_{0i}^T\mathbf{y}_{0i})\ ;$$

or

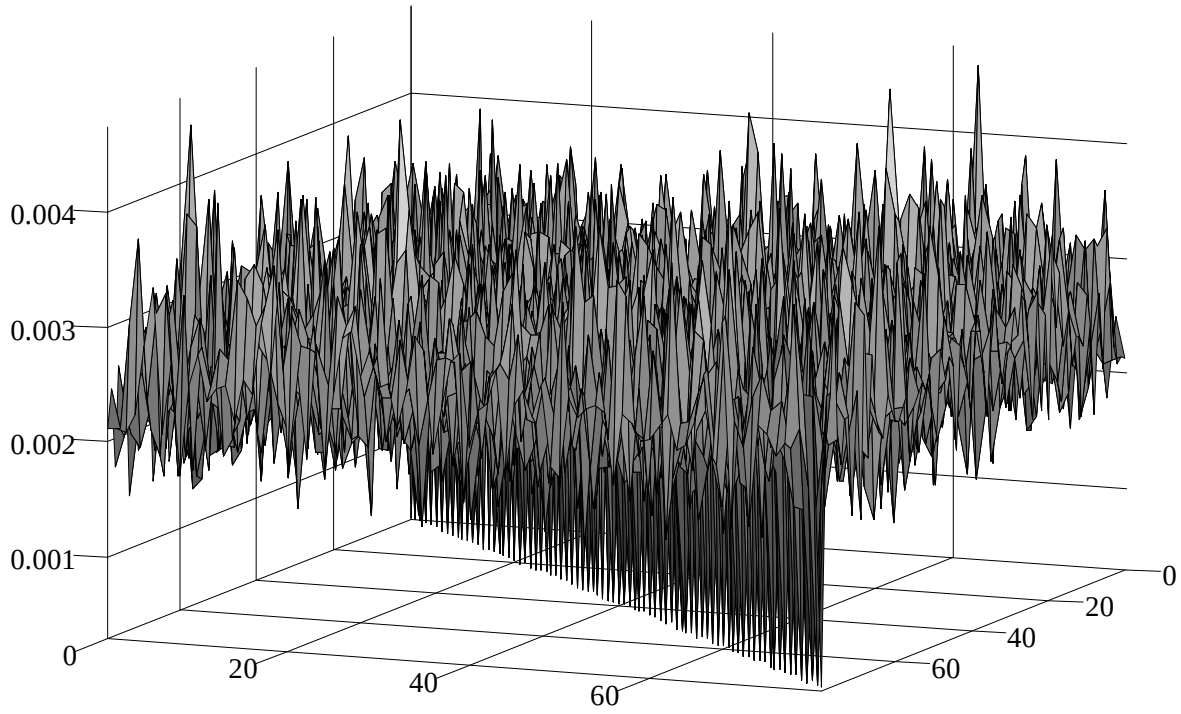
$$(1 - \cos(\mathbf{L}_{ij})) \rightarrow \min \quad \text{as } i = j\ ,\ \text{as } \mathbf{L}_{ij} < 90°.$$

In the figures enclosed you can see results of computer modeling.

Figure 20 shows the development of magnitudes (1 - cos ($\mathbf{L}_{ij}$)), which were averaged by 120 realizations (the X - axes represents the number of a vector from the reference array, which was sequentially compared with all vectors of the sample matrix, on Y - axes - the number of a vector from the sample array). Figure 21 represents values of the diagonal elements of matrix Numb3_1 (see fig. 20). The figure shows

that results of modeling, which obtained by one vector from the array $\mathbf{Z}_0$, will not essentially change if we take any other vector from $\mathbf{Z}_0$ as reference (only for the type of image we consider). For the computing we chose the vector $\mathbf{y}_{0\,40}$.

On the figure 22 the dependence of (1 - cos ($\mathbf{L}_{ij}$)) on root mean square deviation of additive noise (Nsigm3 was averaged by 120 realizations) is shown. The figures 23, 24 and 25 we can see the sections Stat3_1 and Stat3_2 of the figure Nsigm3, given the root mean square value of noise 3.0 and 15.0 respectively. On the figures 23 and 24 the dot line shows the examples Samp3_1 and Samp3_2 of the realization (1 - cos ($\mathbf{L}_{ij}$)), according to which Stat3_1 and Stat3_2 were calculated. All figures are given in a scale of values $\mathbf{Z}_0$.



Numb3_1

Figure 20.

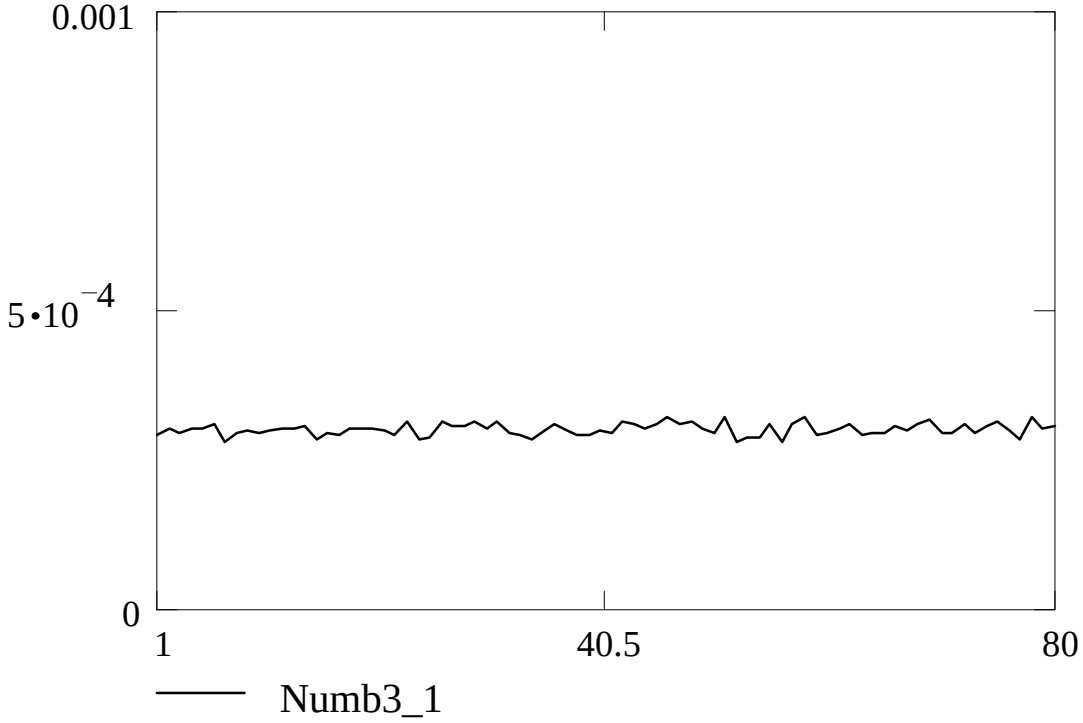


Figure 21.

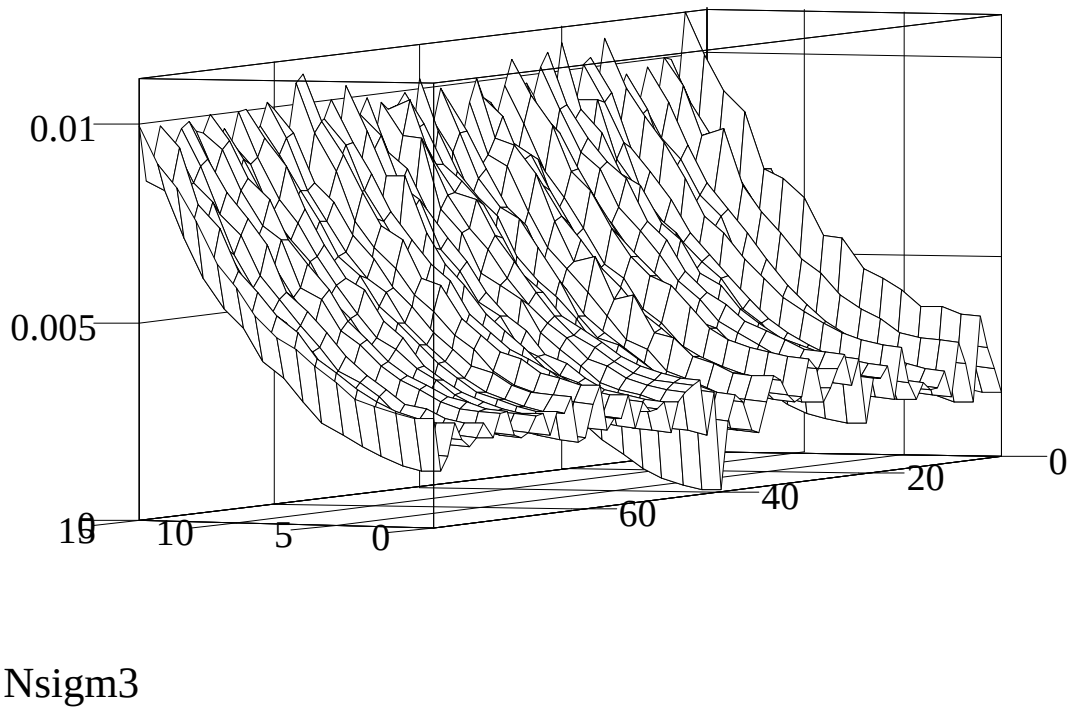


Figure 22.

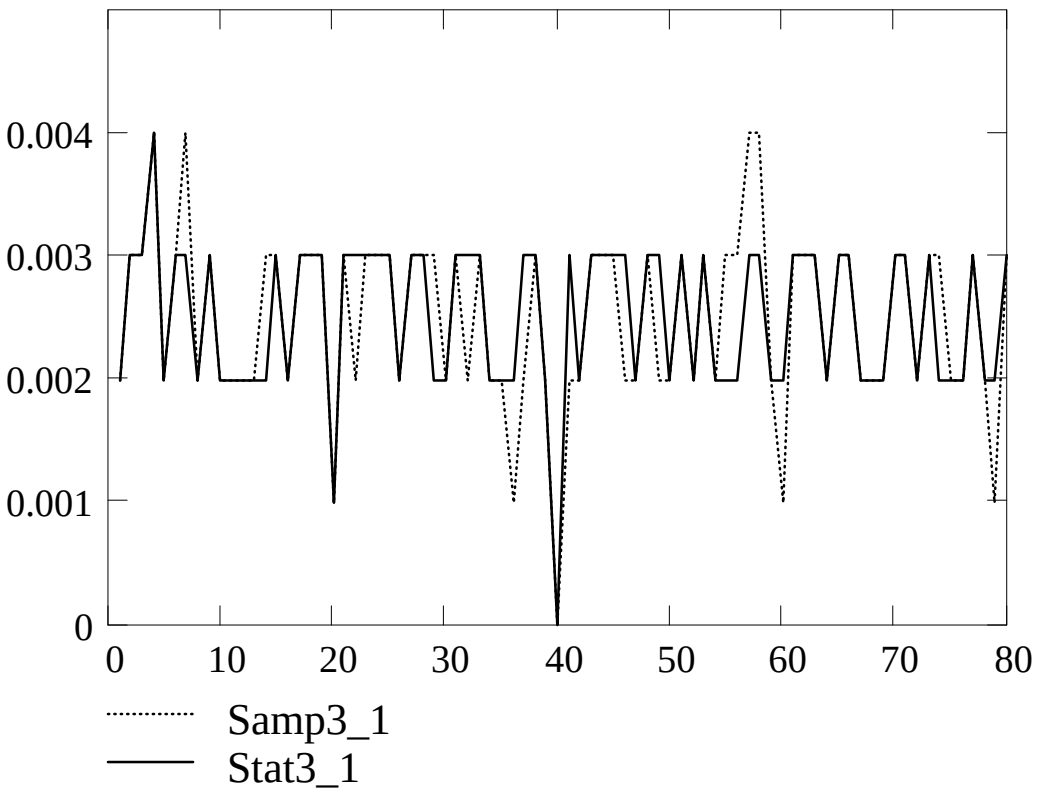


Figure 23.

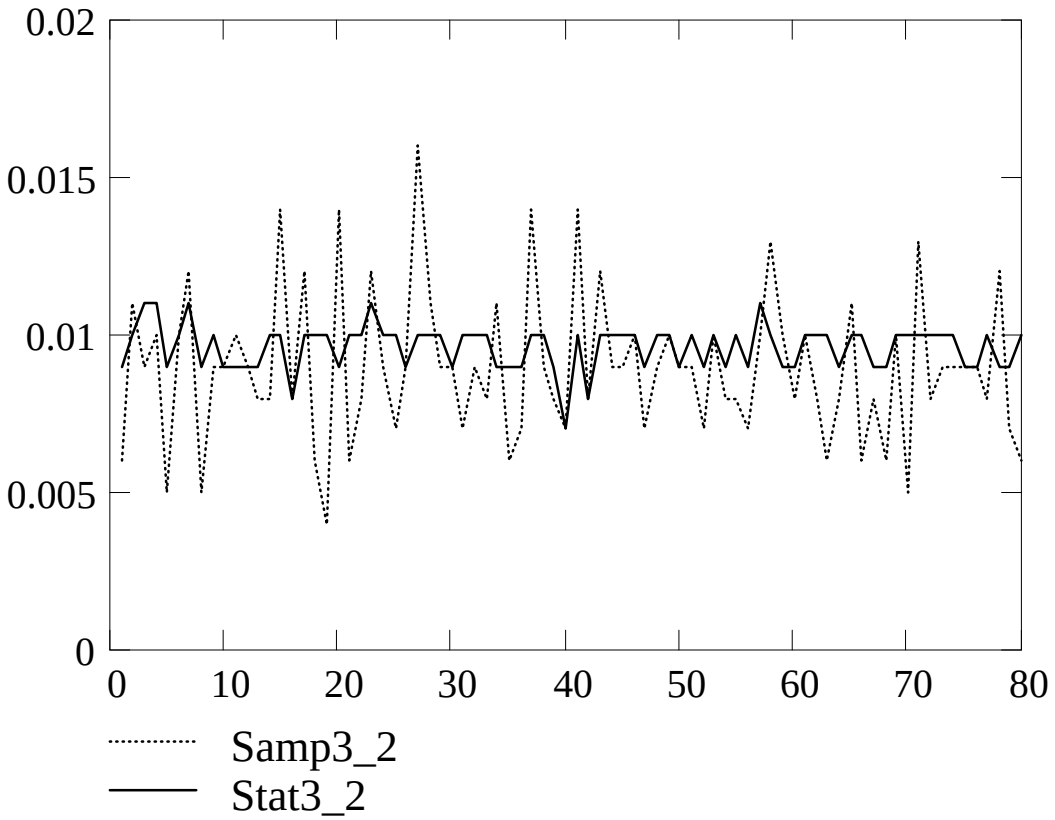


Figure 24.

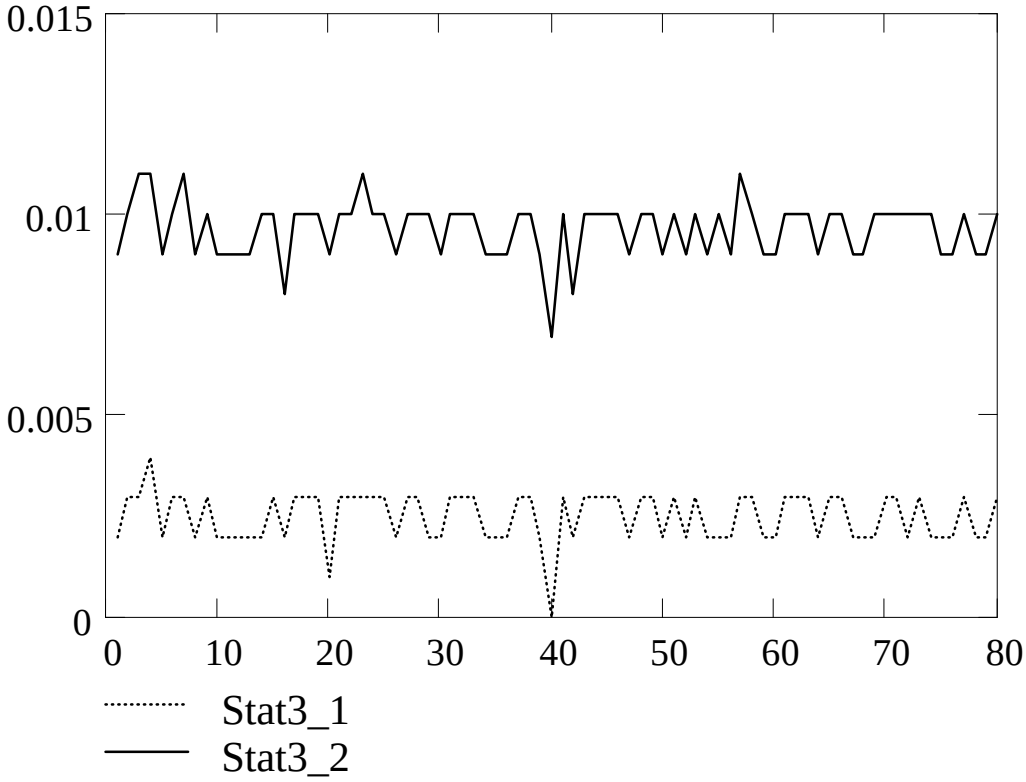


Figure 25.

4. The comparative analysis of the considered criteria.

For comparison of criteria effectiveness we calculated values according to formulas (19) and (20). The calculated values for three criteria are shown on figures 26 and 27. Axes X show rations of root mean square deviation of the noise and reference arrays and represents noise / signal ratio.

$$RMS_j := \sqrt{\frac{1}{(78)} \cdot \left[\sum_{i=1}^{39} \left(mean_j - Nsigm_{j,i}\right)^2 + \sum_{i=41}^{80} \left(mean_j - Nsigm_{j,i}\right)^2 \right]} \quad (19)$$

$$Critery_j := \frac{\left(mean_j - Nsigm_{j,40}\right)}{\sqrt{\frac{1}{(78)} \cdot \left[\sum_{i=1}^{39} \left(mean_j - Nsigm_{j,i}\right)^2 + \sum_{i=41}^{80} \left(mean_j - Nsigm_{j,i}\right)^2 \right]}}, \quad (20)$$

where $mean_j$ - the average value, which is calculated according to formula:

$$mean_j := \frac{1}{79} \cdot \left(\sum_{i=1}^{39} Nsigm_{j,i} + \sum_{i=41}^{80} Nsigm_{j,i} \right).$$

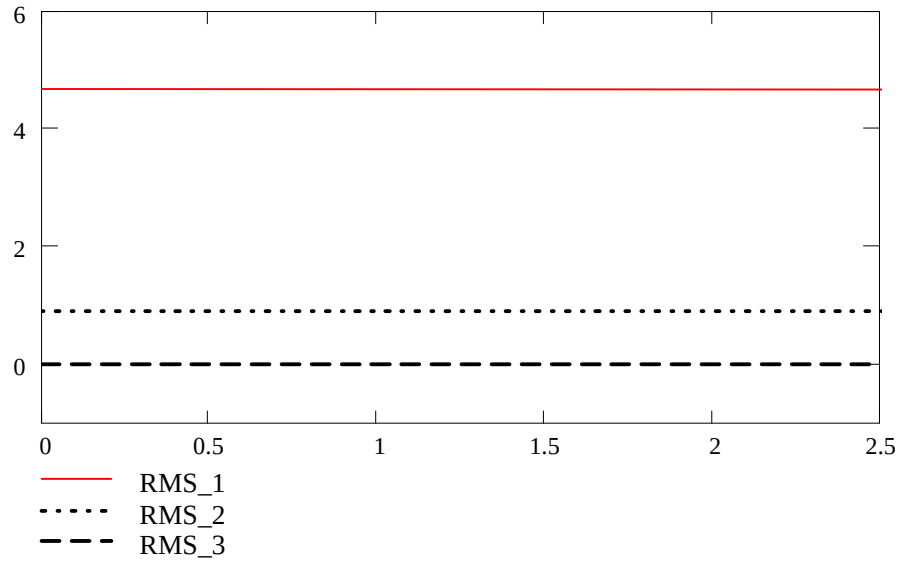


Figure 26.

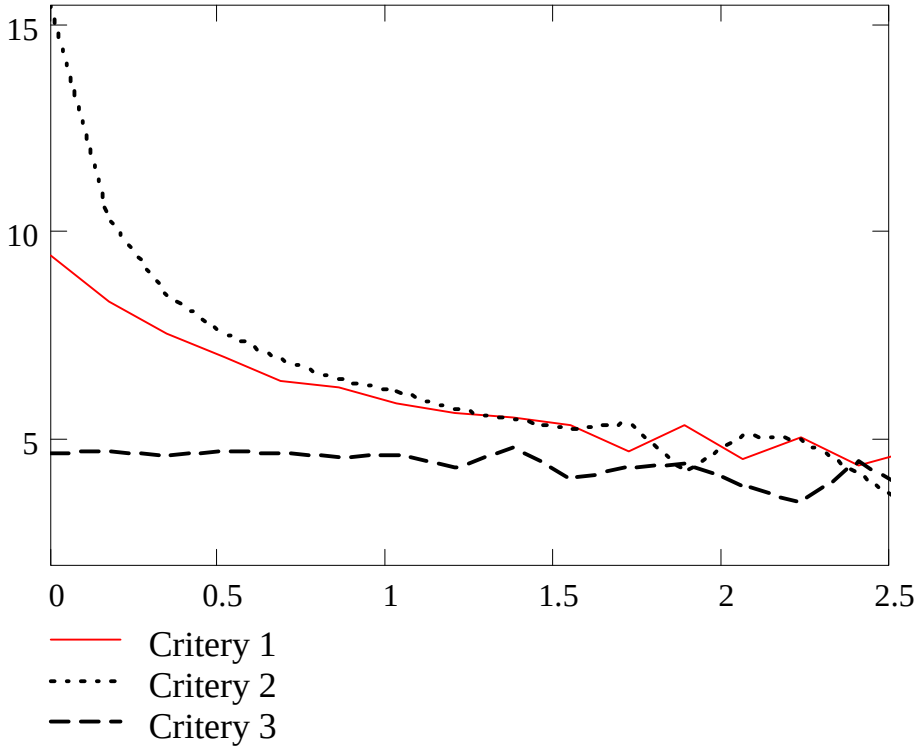


Figure 27.

RESULTS

1. The developed mathematical model visually enables to describe possible (existing) errors in systems, operating with dynamic images (without rotation).
2. This mathematical model can be applied to the analysis of various algorithms for processing two-dimensional arrays of data. For example, in radar-tracking systems or/and quality control systems.
3. This mathematical model is not equally effective for one-dimensional and two-dimensional arrays. In processing for one-dimensional arrays [3] it does not give advantage in comparison with other methods of sample array distortions simulation.
4. In this research we compared three criteria: minimum norm of difference vector; minimum sum of deviation; minimum angle deviation.
5. Standard deviation of vectors values, which corresponds to the distance between vectors of reference and sample arrays, practically does not depend on the ratio of root mean square deviations of noise array and reference array. It is true for all three criteria.
6. The minimum sum of deviation criterion has the best selective ability comparing to minimum norm of difference vector and minimum angle of deviation criteria. It is true for images with a casual picture character and if ratio of root mean square deviations $\mathbf{N}$ and $\mathbf{Z}_0$ is less than 1,25.
7. Minimum sum of deviation criterion requires smaller number of calculator elementary operation, than two other criteria. Therefore, this criterion is more suitable for computing systems making decisions in real time.

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3. Ñ. Êîò "Ìåòîä èäåíòèôèêàöèè áèîýëåêòðè÷åñêèõ ñèãíàëîâ ñ èñïîëüçîâàíèåì ðàçëîæåíèÿ â ñîáñòâåííî ìáçèñå". Òåçèñû äîêëàäîâ. Ïåðâàÿ ìåæäóíàðîäíàÿ ìîëîäåæíàÿ øêîëà ñåìèíàð "ÁÈÊÀÌ Ï-98", ÑÏá., ÃÓÀÏ, 22-26 èþëÿ 1998, ñ. 23-24.